3066. *Proposed by Gabriel Dospinescu, Onesti, Romania.*

Given an integer $n > 2$, let $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ be subsets of $S = \{1, 2, \ldots, n\}$ with the property that for all $i, j \in S$, the subsets A_i and B_j have exactly one element in common. Prove that, if there are at least two distinct subsets among $B_1, B_2, \ldots, B_n$, then there exists a non-empty subset $T \subseteq S$ that has an even number of elements in common with each of the subsets $A_1, A_2, \ldots, A_n$.

3067. *Proposed by Gabriel Dospinescu, Onesti, Romania.*

Find all functions $f : (0,\infty) \to (0,\infty)$ such that

1. $f\big(f(f(x))\big) + 2x = f(3x)$ for all $x > 0$, and
2. $\lim\limits_{x\to\infty}\big(f(x) - x\big) = 0$.

3068. *Proposed by Vasile Cîrtoaje, University of Ploiesti, Romania.*

Let a, b, c be non-negative real numbers, no two of which are zero. Prove that

$$\sqrt{1+\frac{48a}{b+c}} + \sqrt{1+\frac{48b}{c+a}} + \sqrt{1+\frac{48c}{a+b}} \;\geq\; 15\,,$$

and determine when there is equality.

3069. *Proposed by Cristinel Mortici, Valahia University of Targoviste, Romania.*

Let $A, B \in M_2(\mathbb{C})$ be such that $(AB)^2 = A^2B^2$. Prove that

$$\det(I + AB - BA) \;=\; 1\,.$$

3070. *Proposed by Zhang Yun, High School attached to Xi′ An Jiao Tong University, Xi′ An City, Shan Xi, China.*

Let $x_1, x_2, \ldots, x_n$ be positive real numbers such that

$$x_1 + x_2 + \cdots + x_n \;\geq\; x_1x_2\cdots x_n\,.$$

Prove that

$$(x_1x_2\cdots x_n)^{-1}\left(x_1^{n-1} + x_2^{n-1} + \cdots + x_n^{n-1}\right) \;\geq\; \sqrt[n-1]{n^{n-2}}\,,$$

and determine when there is equality.

3071. *Proposed by Arkady Alt, San Jose, CA, USA.*

Let $k > -1$ be a fixed real number. Let a, b, and c be non-negative real numbers such that $a + b + c = 1$ and $ab + bc + ca > 0$. Find

$$\min\left\{\frac{(1+ka)(1+kb)(1+kc)}{(1-a)(1-b)(1-c)}\right\}\,.$$