$$\iff \quad \sum_{cyc} \frac{1}{6-x^2} \le \frac{4\sqrt{3}}{7xyz}.$$

$$\frac{1}{6-x^2} + \frac{1}{6-y^2} + \frac{1}{6-z^2} \quad \le \quad \frac{4\sqrt{3}}{7xyz}.$$

Remark: Since $\frac{4\sqrt{3}}{7} < 1$ we have $\frac{1}{6-x^2} + \frac{1}{6-y^2} + \frac{1}{6-z^2} \le \frac{4\sqrt{3}}{7xyz} < \frac{1}{xyz}$.

So, the inequality in the formulation of problem could have been stated with the stronger statement

$$\frac{1}{6-x^2} + \frac{1}{6-y^2} + \frac{1}{6-z^2} \quad \le \quad \frac{4\sqrt{3}}{7xyz},$$ instead of with the weaker one of

$$\frac{1}{6-x^2} + \frac{1}{6-y^2} + \frac{1}{6-z^2} \quad \le \quad \frac{1}{xyz}.$$

* *Editor's comment:* The inequality $q \le \dfrac{p^2}{3}$ is equivalent to $3abc(a+b+c) \le (ab+bc+ca)^2$ which is equivalent to $abc(a+b+c) \le a^2b^2+b^2c^2+c^2a^2$ which is implied by adding up $a^2bc \le 0.5a^2(b^2+c^2)$ and its cyclic variants.

Also solved by Kee-Wai Lau*, Hong Kong, China; Ecole Suppa, Teramo, Italy; Albert Stadler*, Herrliberg, Switzerland; Titu Zvonaru, Comănesti, Romania jointly with Neculai Stanciu, Buzău, Romania, and the proposer. (* Observed, specifically stated and proved the stricter inequality.)

- **5198:** *Proposed by José Luis Díaz-Barrero, Barcelona, Spain*

Let m, n be positive integers. Calculate,

$$\sum_{k=1}^{2n} \prod_{i=0}^{m} \left(\lfloor \frac{k+1}{2} \rfloor + a + i \right)^{-1},$$

where a is a nonnegative number and $\lfloor x \rfloor$ represents the greatest integer less than or equal to x.

Solution 1 by Arkady Alt, San Jose, CA

$$\sum_{k=1}^{2n} \prod_{i=0}^{m} \left(\left\lfloor \frac{k+1}{2} \right\rfloor + a + i \right)^{-1}$$

$$= \sum_{k=1}^{n} \prod_{i=0}^{m} \left(\left\lfloor \frac{2k-1+1}{2} \right\rfloor + a + i \right)^{-1} + \sum_{k=1}^{n} \prod_{i=0}^{m} \left(\left\lfloor \frac{2k+1}{2} \right\rfloor + a + i \right)^{-1}$$

$$= 2 \sum_{k=1}^{n} \prod_{i=0}^{m} (k+a+i)^{-1}$$

$$= \quad 2\sum_{k=1}^{n} \frac{1}{(k+a)(k+1+a)\ldots(k+m+a)}$$

$$= \quad \frac{2}{m}\sum_{k=1}^{n}\left(\frac{1}{(k+a)(k+1+a)\ldots(k+m-1+a)} - \frac{1}{(k+1+a)(k+2+a)\ldots(k+m+a)}\right)$$

$$= \quad \frac{2}{m}\left(\frac{1}{(1+a)(2+a)\ldots(m+a)} - \frac{1}{(n+1+a)(n+2+a)\ldots(n+m+a)}\right).$$

Solution 2 by Anastasios Kotronis, Athens, Greece

By a direct calculation, using the identity $\Gamma(x+1) = x\Gamma(x), \quad x > 0$ for the Γ function, we can see that

$$\prod_{i=0}^{m} \frac{1}{b+i} = \frac{\Gamma(b)}{\Gamma(b+m+1)} = \frac{1}{m}\left(\frac{\Gamma(b)}{\Gamma(b+m)} - \frac{\Gamma(b+1)}{\Gamma(b+m+1)}\right) \qquad b > 0. \qquad (1)$$

Now

$$\sum_{k=1}^{2n}\prod_{i=0}^{m}\left(\left[\frac{k+1}{2}\right] + a + i\right)^{-1}$$

$$= \quad \sum_{k=1,3,\ldots,2n-1}\prod_{i=0}^{m}\left(\frac{k+1}{2} + a + i\right)^{-1} + \sum_{k=2,4,\ldots,2n}\prod_{i=0}^{m}\left(\frac{k}{2} + a + i\right)^{-1}$$

$$= \quad 2\sum_{k=1}^{n}\prod_{i=0}^{m}(k+a+i)^{-1}$$

$$\overset{(1)}{=} \quad \frac{2}{m}\sum_{k=1}^{n}\left(\frac{\Gamma(a+k)}{\Gamma(a+k+m)} - \frac{\Gamma(a+k+1)}{\Gamma(a+k+m+1)}\right)$$

$$= \quad \frac{2}{m}\left(\frac{\Gamma(a+1)}{\Gamma(a+1+m)} - \frac{\Gamma(a+n+1)}{\Gamma(a+n+m+1)}\right).$$

Also solved by Albert Stadler, Herrliberg, Switzerland and the proposer.

- **5199:** *Proposed by Ovidiu Furdui, Cluj, Romania*

Let $k > 0$ and $n \geq 0$ be real numbers. Calculate,

$$\int_0^1 x^n \ln\left(\sqrt{1+x^k} - \sqrt{1-x^k}\right) dx.$$

Solution by Anastasios Kotronis, Athens, Greece