S276. Let a, b, c be real numbers such that

$$\frac{2}{a^2+1}+\frac{2}{b^2+1}+\frac{2}{c^2+1}\geq 3.$$

Prove that $(a-2)^2+(b-2)^2+(c-2)^2\geq 3$.

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Notice that

$$\sum_{cyc}\frac{2}{a^2+1}\geq 3 \iff \sum_{cyc}\left(\frac{2}{a^2+1}-1\right)\geq 0 \iff \sum_{cyc}\frac{1-a^2}{1+a^2}\geq 0.$$

Now, the key part is to see that

$$\begin{aligned}(a-2)^2-\frac{2}{a^2+1} &= \frac{a^4-4a^3+5a^2-4a+2}{a^2+1}=\frac{a^4-4a^3+6a^2-4a+1+\left(1-a^2\right)}{a^2+1}\\ &= \frac{(a-2)^4}{a^2+1}+\frac{1-a^2}{1+a^2}.\end{aligned}$$

It follows that

$$\sum_{cyc}(a-2)^2=\sum_{cyc}\frac{2}{a^2+1}+\sum_{cyc}\frac{(a-2)^4}{a^2+1}+\sum_{cyc}\frac{1-a^2}{1+a^2}\geq\sum_{cyc}\frac{2}{a^2+1}\geq 3.$$

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