O190. Let ABC be a triangle with sidelengths a, b, c and medians m_a, m_b, m_c. Prove that

$$m_a + m_b + m_c \leq \frac{1}{2}\sqrt{7(a^2+b^2+c^2)+2(ab+bc+ca)}.$$

Proposed by Dorin Andrica, Babes-Bolyai University, Cluj-Napoca, Romania

Solution by Arkady Alt, San Jose, California, USA

Since

$4m_bm_c = \sqrt{\left(2\left(a^2+c^2\right)-b^2\right)\left(2\left(a^2+b^2\right)-c^2\right)} = \sqrt{4a^4+2a^2\left(b^2+c^2\right)+5b^2c^2-2b^4-2c^4} =$
$\sqrt{4a^4+2a^2\left(b^2+c^2\right)+b^2c^2-2\left(b^2-c^2\right)^2} = \sqrt{\left(2a^2+bc\right)^2-2\left(\left(b^2-c^2\right)^2-a^2\left(b-c\right)^2\right)} \leq$
$\sqrt{\left(2a^2+bc\right)^2-2\left(b-c\right)^2\left(a+b+c\right)\left(b+c-a\right)} \leq \sqrt{\left(2a^2+bc\right)^2} = 2a^2+bc$ and
$4\left(m_a^2+m_b^2+m_c^2\right) = 3\sum_{cyc} a^2$ then $4\left(m_a+m_b+m_c\right)^2 = 3\sum_{cyc} a^2 + 8\sum_{cyc} m_bm_c \leq$

$$3\sum_{cyc} a^2 + 2\sum_{cyc}\left(2a^2+bc\right) = 7\sum_{cyc} a^2 + 2\sum_{cyc} bc.$$

Also solved by Daniel Lasaosa, Universidad Pública de Navarra, Spain; Samer SerajOntario, Canada; Scott H. Brown, Auburn University, USA.