J460. Prove that for all positive real numbers x, y, z

$$\left(x^3+y^3+z^3\right)^2 \geq 3\left(x^2y^4+y^2z^4+z^2x^4\right).$$

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By AM-GM Inequality

$$\left(x^3+y^3+z^3\right)^2 = x^6+y^6+z^6+2x^3y^3+2y^3z^3+2z^3x^3 =$$

$$\sum_{cyc}\left(x^6+2z^3x^3\right) \geq \sum_{cyc} 3\sqrt[3]{x^6\cdot\left(z^3x^3\right)^2} = \sum_{cyc} 3\sqrt[3]{x^{12}z^6} =$$

$$3\sum_{cyc} x^4z^2 = \quad 3\left(x^2y^4+y^2z^4+z^2x^4\right)$$

and the conclusion follows.

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