J437. Let a, b, c be real numbers such that $\left(a^2+2\right)\left(b^2+2\right)\left(c^2+2\right)=512$. Prove that $|ab+bc+ca| \leq 18$.

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Solution by Arkady Alt, San Jose, CA, USA

Since $\left(a^2+2\right)\left(b^2+2\right)\left(c^2+2\right)=(abc-2(a+b+c))^2+2(ab+bc+ca-2)^2$ then

$2(ab+bc+ca-2)^2 \leq 512 \iff |ab+bc+ca-2| \leq 16$.

Also we have $\quad |ab+bc+ca-2|+2 \geq .|ab+bc+ca-2+2|=|ab+bc+ca|$

Hence, $|ab+bc+ca| \leq 18$.

Also solved by Polyahedra, Polk State College, USA; Naïm Mégarbané, UPMC, Paris, France; Nikos Kalapodis, Patras, Greece; Nermin Hodžic, Dobošnica, Bosnia and Herzegovina; AN-anduud Problem Solving Group, Ulaanbaatar, Mongolia; Nguyen Ngoc Tu, Ha Giang, Vietnam; Nicusor Zlota, Traian Vuia Technical College, Focsani, Romania; Paolo Perfetti, Università degli studi di Tor Vergata Roma, Rome, Italy; Kevin Soto Palacios, Huarmey, Perú; Titu Zvonaru, Comănești, Romania.