J311. Let a, b, c be real numbers greater than or equal to 1. Prove that

$$\frac{a\left(b^2+3\right)}{3c^2+1}+\frac{b\left(c^2+3\right)}{3a^2+1}+\frac{c\left(a^2+3\right)}{3b^2+1}\geq 3.$$

Proposed by Titu Andreescu, University of Texas at Dallas, USA

Solution by Arkady Alt, San Jose, CA, USA

By AM-GM Inequality

$$\sum_{cyc}\frac{a(b^2+3)}{3c^2+1}\geq 3\sqrt[3]{\prod_{cyc}\frac{a(b^2+3)}{3c^2+1}}=3\sqrt[3]{\prod_{cyc}\frac{a(a^2+3)}{3a^2+1}}\geq 3$$

since

$$\frac{a(a^2+3)}{3a^2+1}\geq 1 \iff (a-1)^3\geq 0.$$

Also solved by Bedri Hajrizi, Gjimnazi "Frang Bardhi", Mitrovicë, Kosovë; Daniel Lasaosa, Pamplona, Spain; Jaesung Son, Ridgewood, NJ, USA; Jongyeob Lee, Stuyvesant High School, NY, USA; Yeonjune Kang, Peddie School, Hightstown, NJ, USA; William Kang, Bergen County Academies, Hackensack, NJ, USA; Chaeyeon Oh, Episcopal High School, Alexandra, VA, USA; Kyoung A Lee, The Hotchkiss School, Lakeville, CT, USA; Seung Hwan An, Taft School, Watertown, CT, USA; Polyahedra, Polk State College, USA; Alyssa Hwang, Kent Place School Summit, NJ, USA; Woosung Jung, Korea International School, South Korea; Daniel Jhiseung Hahn, Phillips Exeter Academy, Exeter, NH, USA; Seong Kweon Hong,The Hotchkiss School, Lakeville, CT; Michael Tang, Edina High School, MN, USA; Adnan Ali, A.E.C.S-4, Mumbai, India; Bodhisattwa Bhowmik, RKMV, Agartala, Tripura, India; Ilyes Hamdi, Lycée Voltaire, Doha, Qatar; Prithwijit De, HBCSE, Mumbai, India; Alessandro Ventullo, Milan, Italy; Zachary Chase, University School of NSU, FL, USA; George Gavrilopoulos, Nea Makri High School, Athens, Greece; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Florin Stanescu, Cioculescu Serban High School, Gaesti, Romania; Radouan Boukharfane, Sidislimane, Morocco; Paolo Perfetti, Università degli studi di Tor Vergata Roma, Roma, Italy; Moubinool Omarjee, Lycée Henri IV, Paris, France.