

## Junior problems

J265. Let  $a, b, c$  be real numbers such that

$$5(a + b + c) - 2(ab + bc + ca) = 9.$$

Prove that any two of the equalities

$$|3a - 4b| = |5c - 6|, \quad |3b - 4c| = |5a - 6|, \quad |3c - 4a| = |5b - 6|$$

imply the third.

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Since

$$\begin{aligned} & (3a - 4b)^2 - (5c - 6)^2 + (3b - 4c)^2 - (5a - 6)^2 + (3c - 4a)^2 - (5b - 6)^2 = \\ & = 60(a + b + c) - 24(ab + ac + bc) - 108 = 12(5(a + b + c) - 2(ab + bc + ca) - 9) = 0 \end{aligned}$$

then from any two equalities, let it be  $\begin{cases} |3a - 4b| = |5c - 6| \\ |3b - 4c| = |5a - 6| \end{cases} \iff \begin{cases} (3a - 4b)^2 = (5c - 6)^2 \\ (3b - 4c)^2 = (5a - 6)^2 \end{cases}$

immediately yields

$$(3c - 4a)^2 - (5b - 6)^2 = 0 \iff |3c - 4a| = |5b - 6|.$$

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