Junior problems

J199. Prove that there are infinitely many pairs (p,q) of primes such that $p^6 + q^4$ has two positive divisors whose difference is 4pq.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

Solution by Arkady Alt, San Jose, USA

Let p=2 and let q be odd prime number. Then

$$p^6 + q^4 = 64 + q^4 = (8 + 4q + q^2)(8 - 4q + q^2)$$

and we have

$$(8+4q+q^2) - (8-4q+q^2) = 8q = 4pq.$$

Also solved by Daniel Lasaosa, Universidad Pública de Navarra, Spain; Prithwijit De, HBCSE, Mumbai, India; Roberto Bosch Cabrera, Havana, Cuba; Tigran Hakobyan, Armenia.