4135. *Proposed by Daniel Sitaru.*

Let ABC be a triangle with $BC = a$, $AC = b$, $AB = c$. Prove that the following relationship holds

$$\sqrt{a} + \sqrt{b} + \sqrt{c} \leq \sqrt{3\Big(\frac{a^2}{b+c-a} + \frac{b^2}{a+c-b} + \frac{c^2}{a+b-c}\Big)}.$$

We received nine solutions. We present the solution by Dionne Bailey, Elsie Campbell and Charles R. Diminnie.

Since $f(x) = \sqrt{x}$ is concave on $(0, \infty)$, Jensen's theorem implies that

$$\sqrt{a} + \sqrt{b} + \sqrt{c} = f(a) + f(b) + f(c) \leq 3f\left(\frac{a+b+c}{3}\right) = \sqrt{3(a+b+c)}. \qquad (1)$$

By the Cauchy-Schwarz inequality, writing $a = \frac{a}{\sqrt{b+c-a}}\sqrt{b+c-a}$ and similarly for b and c, we get

$$a + b + c \leq \Big(\frac{a^2}{b+c-a} + \frac{b^2}{c+a-b} + \frac{c^2}{a+b-c}\Big)^{1/2} (a+b+c)^{1/2},$$

which (dividing both sides by $(a+b+c)^{1/2}$ and multiplying by $\sqrt{3}$) yields

$$\sqrt{3(a+b+c)} \leq \sqrt{3\left(\frac{a^2}{b+c-a} + \frac{b^2}{c+a-b} + \frac{c^2}{a+b-c}\right)}. \qquad (2)$$

Combining (1) and (2), we get the desired inequality; note that equality holds if and only if $a = b = c$, in other words if and only if $\triangle ABC$ is equilateral.

4136. *Proposed by Daniel Sitaru and Mihaly Bencze.*

Prove that if $a, b, c \in (0, \infty)$ then:

$$b\int_0^a e^{-t^2}dt + c\int_0^b e^{-t^2}dt + a\int_0^c e^{-t^2} < \frac{\pi}{2}\sqrt{3(a^2+b^2+c^2)}.$$

We received nine submissions all of which are correct. We present four solutions in all of which we use S to denote the left side of the given inequality.

Solution 1, by Arkady Alt, Sefket Arslanagic, Paul Bracken, and Digby Smith (independently).

Since

$$\int_0^x e^{-t^2}dt \leq \int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$$

for $x = a, b$, and c, we have by the Cauchy-Schwarz Inequality that

$$S \le \sqrt{b^2+c^2+a^2} \cdot \sqrt{\left(\int_0^a e^{-t^2}dt\right)^2 + \left(\int_0^b e^{-t^2}dt\right)^2 + \left(\int_0^c e^{-t^2}dt\right)^2}$$
$$\le \sqrt{a^2+b^2+c^2} \cdot \sqrt{3\left(\frac{\sqrt{\pi}}{2}\right)^2} = \frac{\sqrt{\pi}}{2} \cdot \sqrt{3(a^2+b^2+c^2)}$$
$$< \frac{\pi}{2} \cdot \sqrt{3(a^2+b^2+c^2)}.$$

Solution 2, by Leonard Giugiuc.

As in Solution 1 above, we have

$$\int_0^x e^{-t^2}dt \le \frac{\sqrt{\pi}}{2}$$

for $x = a, b$, and c, so

$$S \le (b+c+a) \cdot \frac{\sqrt{\pi}}{2} < \frac{\pi}{2} \cdot \sqrt{3(a^2+b^2+c^2)}$$

by the AM-QM inequality.

Solution 3, by Leonard Giugiuc.

Since $e^{t^2} = 1 + t^2 + \frac{t^4}{2!} + \cdots$ we have for $x = a, b$, and c,

$$\int_0^x e^{-t^2}dt \le \int_0^x \frac{dt}{1+t^2} = \tan^{-1} x < \frac{\pi}{2}.$$

Hence,

$$S < (a+b+c)\left(\frac{\pi}{2}\right) \le \frac{\pi}{2} \cdot \sqrt{3(a^2+b^2+c^2)},$$

by the AM-QM inequality.

Solution 4, by Kee-Wei Lau.

Using $\int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$, we have

$$S \le (b+c+a)\int_0^\infty e^{-t^2}dt$$
$$= \sqrt{3(a^2+b^2+c^2) - (a-b)^2 - (b-c)^2 - (c-a)^2} \cdot \frac{\sqrt{\pi}}{2}$$
$$< \frac{\pi}{2} \cdot \sqrt{3(a^2+b^2+c^2)}.$$