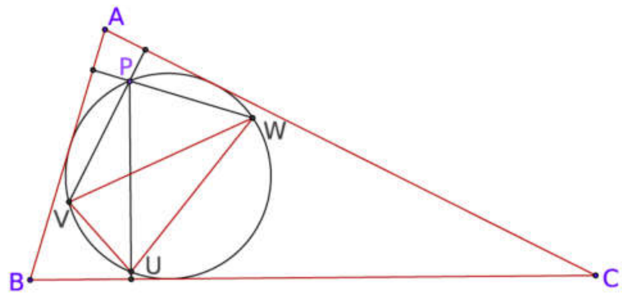

Assume the contrary. Then, there is no loss of generality in assuming that $\hat{U} \neq \hat{A}$. Thus, $\hat{U} = 180° - \hat{A}$.

If $(\hat{V}, \hat{W}) = (\hat{B}, \hat{C})$, then we obtain $180° = \hat{U} + \hat{V} + \hat{W} = 180° - \hat{A} + \hat{B} + \hat{C}$, so that $\hat{A} = \hat{B} + \hat{C} = 90° = 180° - \hat{A} = \hat{U}$, contradicting our assumption that $\hat{U} \neq \hat{A}$. Hence, $(\hat{V}, \hat{W}) \neq (\hat{B}, \hat{C})$. There is no loss of generality in assuming that $\hat{V} = 180° - \hat{B}$. But then, $\hat{U} + \hat{V} = (180° - A) + (180° - B) > 180°$, a contradiction.

Editor's Comments. We were pleased to find that among the six solutions submitted, four different formulas for the area of a triangle were used.

4038. *Proposed by George Apostolopoulos.*

Let x, y, z be positive real numbers such that $x + y + z = xyz$. Find the minimum value of the expression

$$\sqrt{\frac{1}{3}x^4 + 1} + \sqrt{\frac{1}{3}y^4 + 1} + \sqrt{\frac{1}{3}z^4 + 1}.$$

There were 21 correct solutions, with four from one submitter and three from another. An additional solution was incorrect.

Solution 1, by Arkady Alt, Šefket Arslanagić, and Daniel Dan (independently).

Since $xyz = x+y+z \geq 3\sqrt[3]{xyz}$, if follows that $x+y+z = xyz \geq 3\sqrt{3}$. Applying the inequality of the root mean square and arithmetic mean, we have, for $t = x, y, z$,

$$\sqrt{\frac{1}{3}t^4 + 1} = \sqrt{\left(\frac{t^2}{3}\right)^2 + \left(\frac{t^2}{3}\right)^2 + \left(\frac{t^2}{3}\right)^2 + 1}$$
$$\geq \frac{(t^2/3) + (t^2/3) + (t^2/3) + 1}{2} = \frac{t^2 + 1}{2}$$

with equality iff $t = \sqrt{3}$. (Alternatively, the inequality $\sqrt{\frac{1}{3}t^4 + 1} \geq \frac{t^2+1}{2}$ is equivalent to $(t^2 - 3)^2 \geq 0$.) Therefore, the left side of the inequality is not less than

$$\frac{1}{2}(3 + x^2 + y^2 + z^2) \geq \frac{1}{2}\left(3 + \frac{(x+y+z)^2}{3}\right) \geq \frac{1}{2}(3 + (27/3)) = 6.$$

Since equality occurs when $x = y = z = \sqrt{3}$, the desired minimum is 6.