Неравенство
https://www.linkedin.com/groups/8313943/8313943-6385178287785725953
Let x, y and z be positive real numbers. Prove that

$$\sqrt[3]{\frac{x^3+y^3+z^3}{xyz}} + \sqrt{\frac{xy+yz+zx}{x^2+y^2+z^2}} \ge \sqrt[3]{3} + 1.$$

Solution by Arkady Alt , San Jose, California, USA.
Assuming due homogeneity that $x+y+z=1$ and,denoting $p := xy+yz+zx, q := xyz$ we can rewrite inequality of the problem as $\sqrt[3]{\frac{1+3q-3p}{q}} + \sqrt{\frac{p}{1-2p}} \ge \sqrt[3]{3} + 1 \Leftrightarrow$

(**1**) $\sqrt[3]{\frac{1-3p}{q}+3} + \sqrt{\frac{p}{1-2p}} \ge \sqrt[3]{3} + 1.$

Since $3p \le 1$ ($3(xy+yz+zx) \le (x+y+z)^2$) and $q \le \frac{p^2}{3}$ ($3xyz(x+y+z) \le (xy+yz+zx)^2$) we have $\sqrt[3]{\frac{1-3p}{q}+3} + \sqrt{\frac{p}{1-2p}} \ge \sqrt[3]{\frac{1-3p}{p^2/3}+3} + \sqrt{\frac{p}{1-2p}}$.

Let $t := \sqrt{\frac{1-2p}{p}} \ge 1$. Then $p = \frac{1}{t^2+2}$ and $\frac{3(1-3p)}{p^2} + 3 = \frac{1-3\cdot\frac{1}{t^2+2}}{\left(\frac{1}{t^2+2}\right)^2} + 1 =$

$t^2 + t^4 - 1$

Hence $\sqrt[3]{\frac{1-3p}{p^2/3}+3} + \sqrt{\frac{p}{1-2p}} = \sqrt[3]{3t^2+t^4-1} + \frac{1}{t}$ and since*

$\left(\sqrt[3]{3t^2+t^4-1} + \frac{1}{t}\right)' = \frac{4t^3+6t}{3\sqrt[3]{(t^4+3t^2-1)^2}} - \frac{1}{t^2} > 0$ for $t \ge 1$ then

$\sqrt[3]{3t^2+t^4-1} + \frac{1}{t} \ge \sqrt[3]{3\cdot 1^2+1^4-1} + \frac{1}{1} = \sqrt[3]{3} + 1.$

* we have $t^6(4t^3+6t)^3 - 27(t^4+3t^2-1)^2 > t^6(4t^3+4t)^3 - 27(t^4+3t^2)^2 =$
$64t^9(t^2+1)^3 - 27t^4(t^2+3)^2 = 64t^9(t^2+1)^3 - 27t^4(t^2+3)^2 =$
$t^4(64t^{11} + 192t^9 + 192t^7 + 64t^5 - 162t^2 - 27t^4 - 243) =$
$t^4((64t^{11} - 27t^4) + (192t^9 - 162t^2) + (192t^7 + 64t^5 - 243)) > 0.$